Image segmentation

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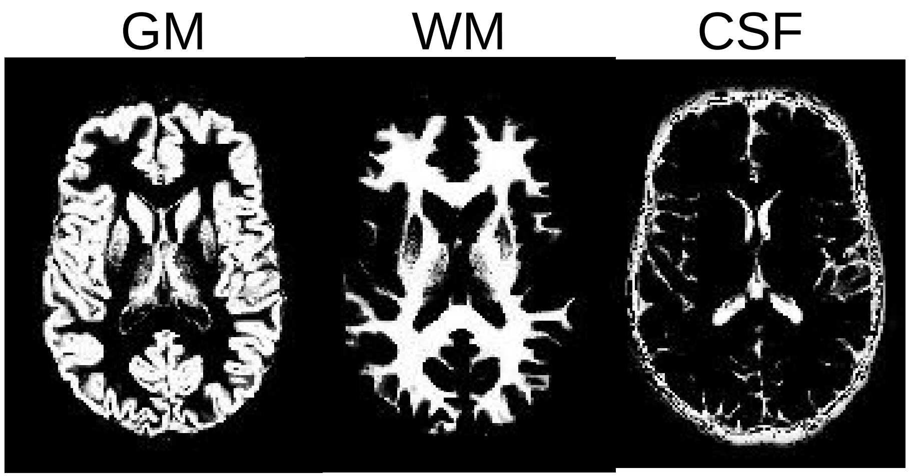
Introduction

- Segmentation is an image analysis task that subdivides an image into disjoint regions of interest for further analysis.

Given Image



Segmented Image

Principal approaches

- Segmentation algorithms generally are based on two basic properties of intensity
  - Discontinuity: finding abrupt changes in intensity (such as edges)
  - Thresholding: partitioning images directly based on intensity

Discontinuity-based method

Review: spatial filtering

- Detection of points, lines, and edges
- This is usually accomplished by applying a suitable mask to the image.

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

$$g(x,y) = \sum_i w_i f_i$$

Detection of discontinuities

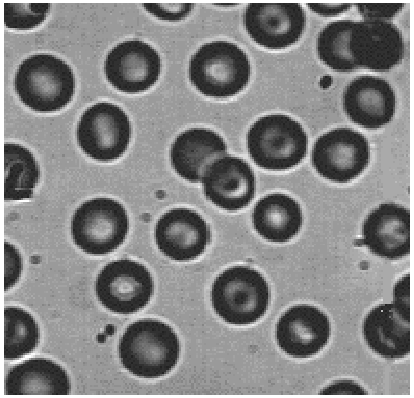
- The gray-level of an isolated point will be very different from its neighbors.
- Can be done by Laplacian 3x3 masks:

$$g(x, y) > T \quad T\text{: pre-defined threshold}$$

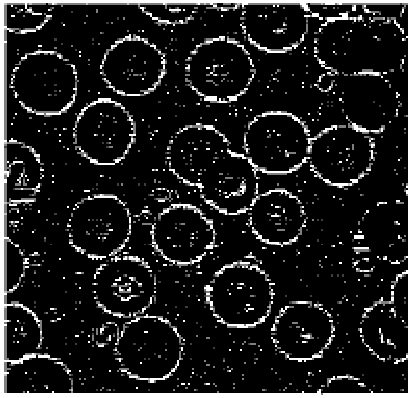
0	1	0
1	-4	1
0	1	0

1	1	1
1	-8	1
1	1	1

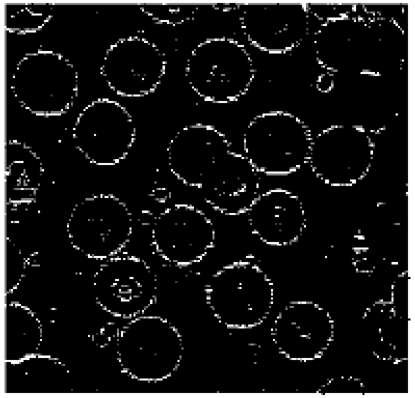
Example: point detection



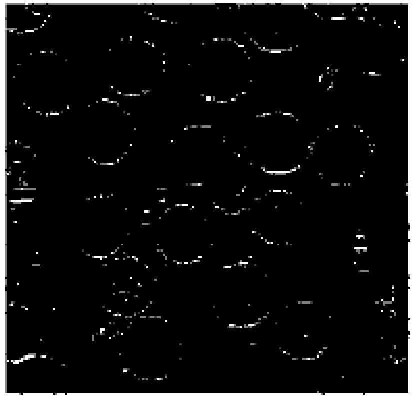
Orig. Image



$T = 0.25$



$T = 0.35$



$T = 0.5$

More: detection of lines

- Apply different masks on the image

-1	-1	-1
2	2	2
-1	-1	-1

horizontal

-1	2	-1
-1	2	-1
-1	2	-1

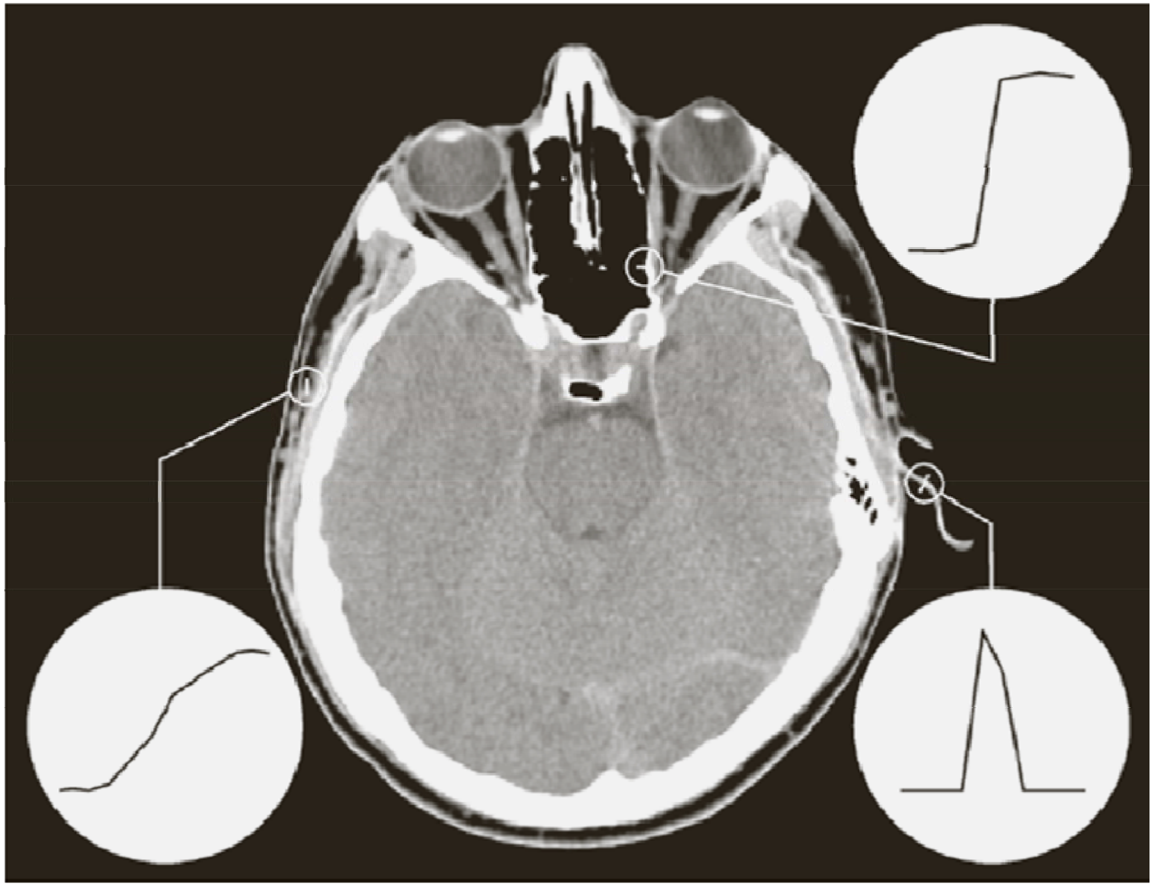
vertical

2	-1	-1
-1	2	-1
-1	-1	2

oblique (+45°)

- Gain information from various masks!

Edges



Step edge

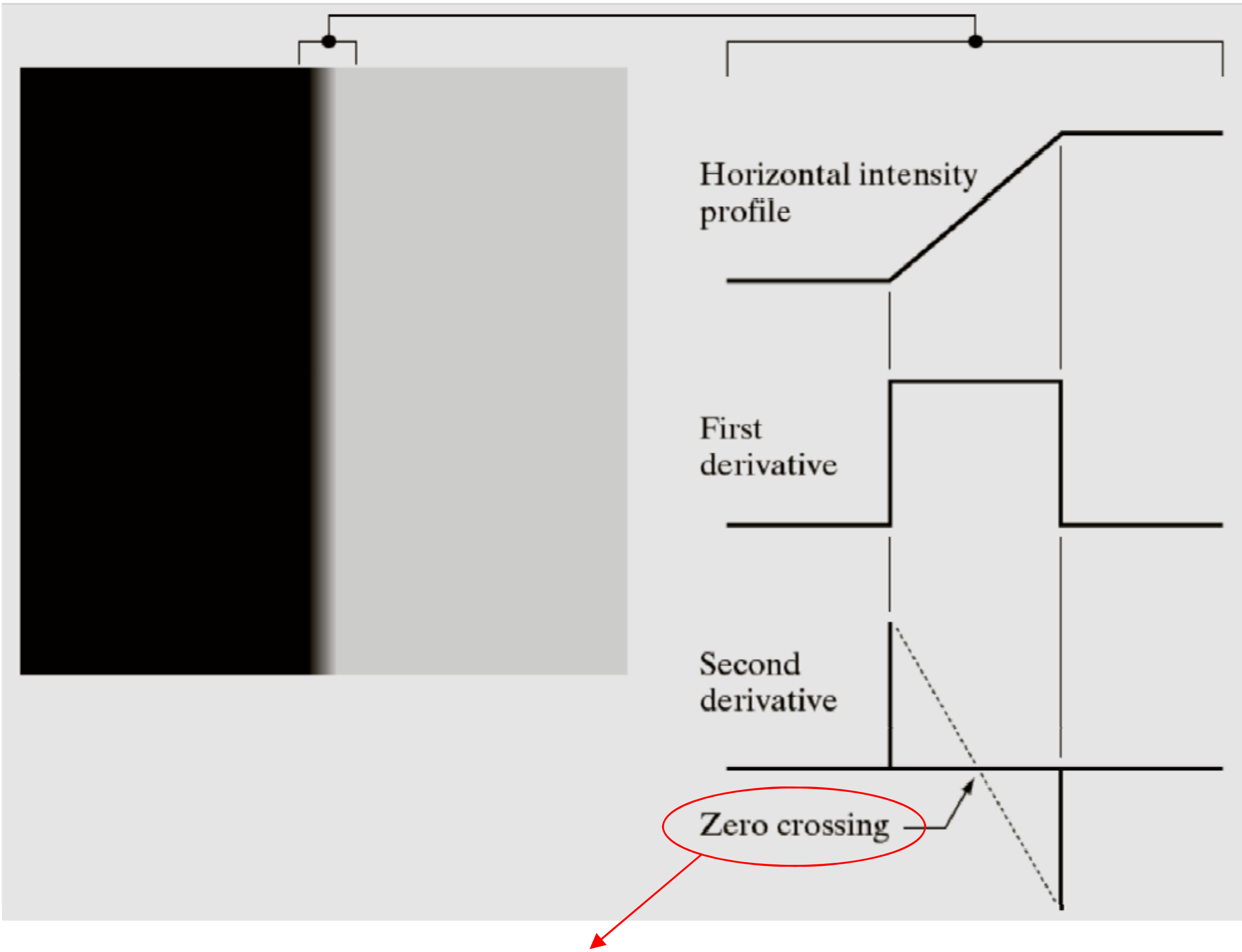
Ramp edge

Roof edge

Edge detection

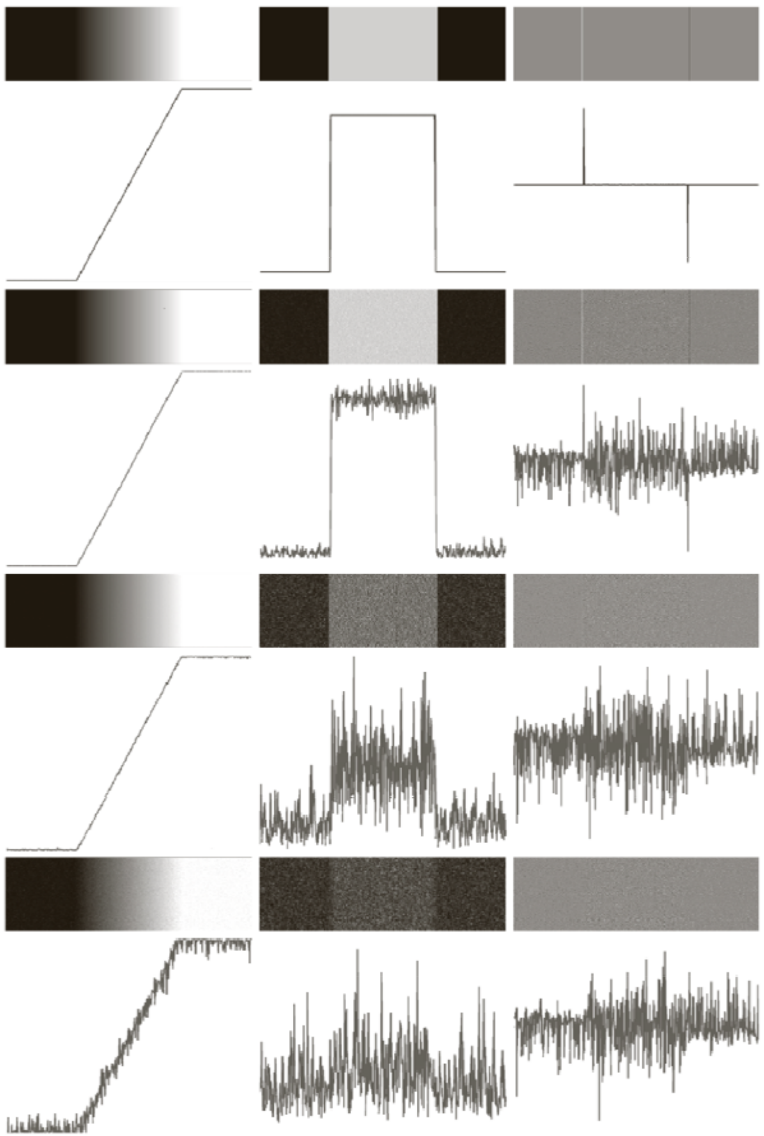
- Most frequently used
- Approaches of edge detection
  - first-order derivative (Gradient operator)
  - second-order derivative (Laplacian operator)
- Edge detection is not that easy since perfect discontinuities seldom exist in real cases.

An ideal vertical ramp edge



Considered the center of a thick edge

Noise corruption



First column (left): images and intensity profiles of a ramp edge corrupted by Gaussian noise of zero mean and standard deviation of 0, 0.1, 1.0, and 10.

Second column (center): first derivatives
Third column (right): second derivatives

Fundamental steps in edge detection

- Prior image smoothing for noise reduction
- Detection of edge points
  - Spatial convolution with suitable kernels
  - Setting threshold for magnitude in 1^{st} derivatives or sign in 2^{nd} derivatives
- Edge localization
  - Only true members are selected from candidates to comprise an edge

Basic edge detection

- Most common differentiation operator is the gradient.

$$\nabla f(x,y) = \begin{bmatrix} \dfrac{\partial f(x,y)}{\partial x} \\ \dfrac{\partial f(x,y)}{\partial y} \end{bmatrix}$$

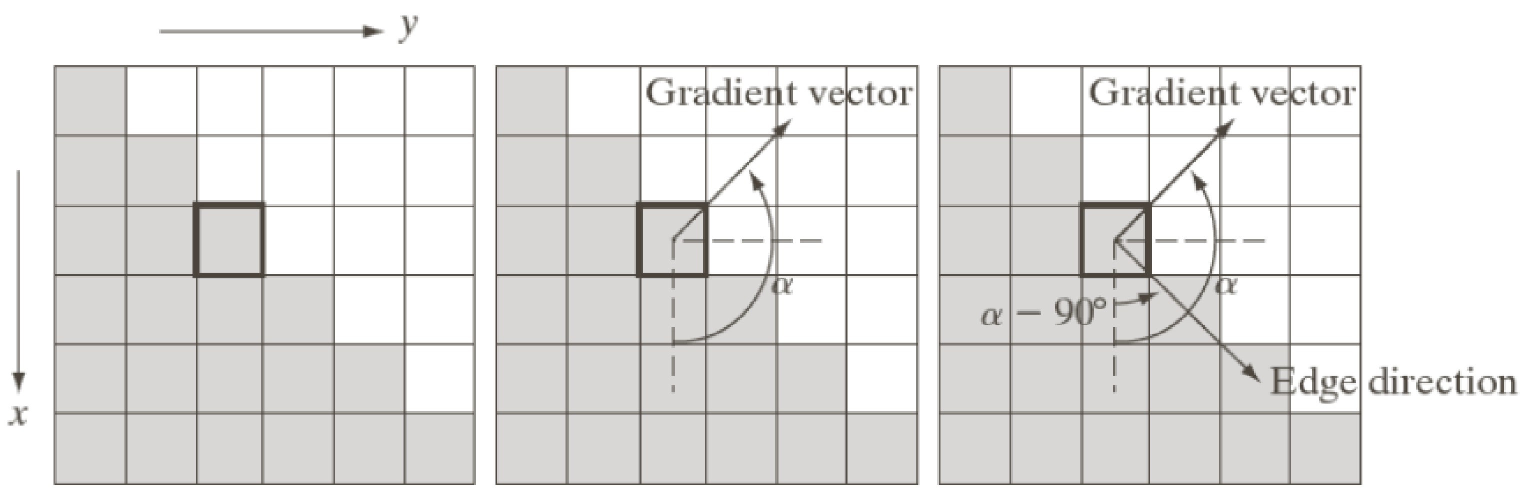
- The <u>magnitude</u> of the gradient:

$$|\nabla f(x,y)| = \left[\left(\frac{\partial f(x,y)}{\partial x} \right)^2 + \left(\frac{\partial f(x,y)}{\partial y} \right)^2 \right]^{1/2}$$

- The <u>direction</u> of the gradient:

$$\angle \nabla f(x,y) = \tan^{-1} \left(\frac{\partial f}{\partial y} \Big/ \frac{\partial f}{\partial x} \right)$$

Edge and the gradient



$$D_h = \frac{1}{2}\left\{\begin{bmatrix}-1 & 1 & 0\end{bmatrix} + \begin{bmatrix}0 & -1 & 1\end{bmatrix}\right\} = \frac{1}{2}\begin{bmatrix}-1 & 0 & 1\end{bmatrix}$$

$$D_v = \frac{1}{2}\left\{\begin{bmatrix}-1\\1\\0\end{bmatrix} + \begin{bmatrix}0\\-1\\1\end{bmatrix}\right\} = \frac{1}{2}\begin{bmatrix}-1\\0\\1\end{bmatrix}$$

More operators of gradients

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

The 3x3 region of an image

−1	0
0	1

0	−1
1	0

Roberts

Roberts cross-gradient operators (1965)
-- diagonal edges

−1	−1	−1
0	0	0
1	1	1

−1	0	1
−1	0	1
−1	0	1

Prewitt

Prewitt operators (1970)
-- symmetric about the center points

−1	−2	−1
0	0	0
1	2	1

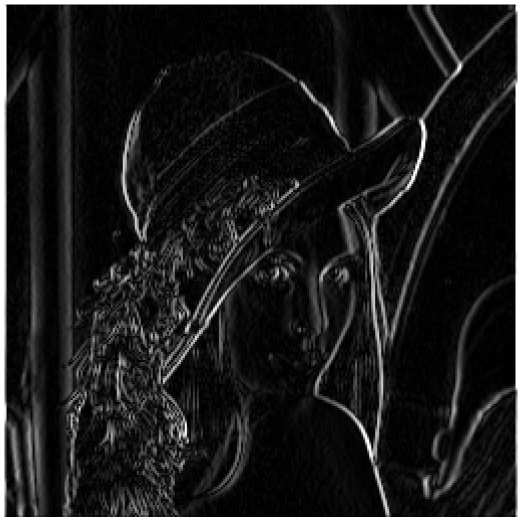
−1	0	1
−2	0	2
−1	0	1

Sobel

Sobel operators (1970)
-- increase center weighting for smoothing

Example: Sobel gradients

Original image (256x256)

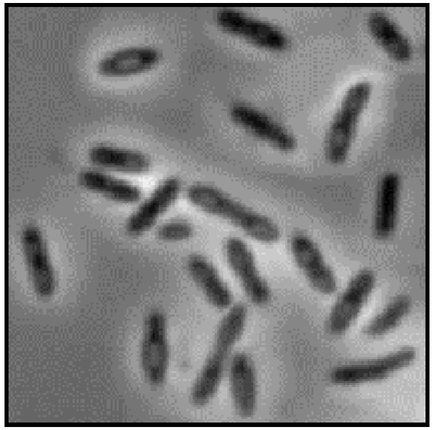


$|f_x|$ (vertical edges)

$|f_y|$ (horizontal edges)

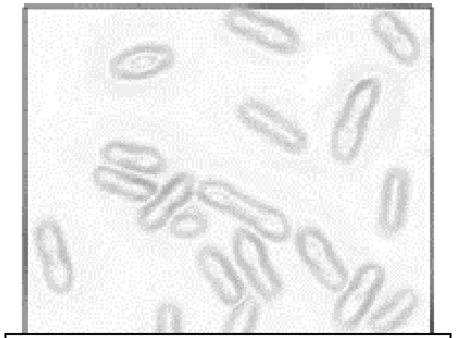
magnitude of f

Example: combining thresholding

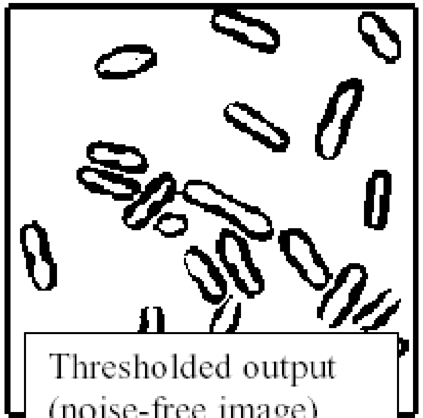


Original image (256x256)

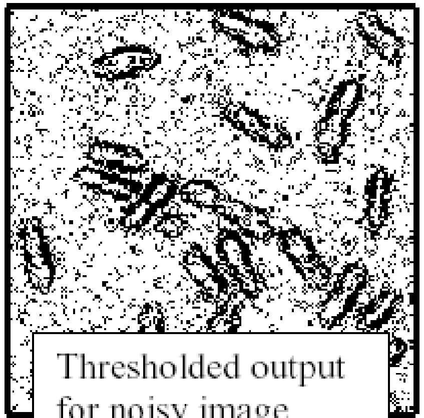
Usual gradient mask



Negative mag. of Gradient (noise-free image)

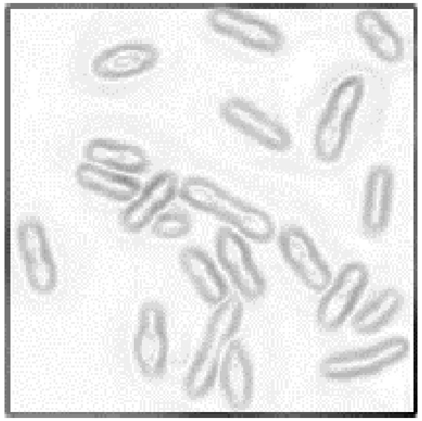


Thresholded output (noise-free image)

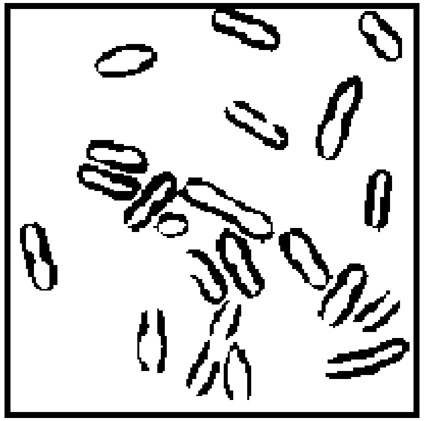


Thresholded output for noisy image

Sobel edge detector



Negative mag. of Gradient (noise-free image)



Thresholded output (noise-free image)

Thresholded output for noisy image

The Laplacian Edge Detector

- The Laplacian of a two-dimensional image $f(x, y)$ is given by

$$\nabla^2 f(x,y) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

- Second derivative
- Rotation invariant
  - Equally sharpen edges in any direction (isotropic)

The Laplacian Edge Detector

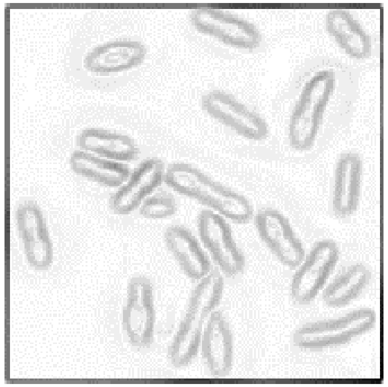
- Approximations of Laplacian

$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix} \quad \begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix} \quad \begin{bmatrix} 1 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 1 \end{bmatrix}$$

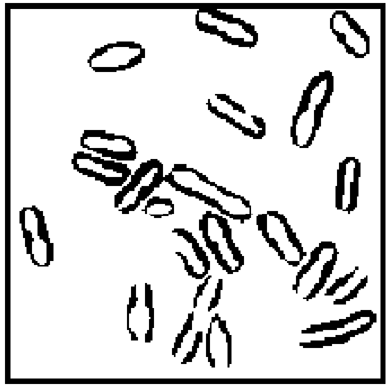
- Properties:
  - Extremely sensitive to noise
  - Double edges detected
  - Useful for gradual transition at edges

Example: Laplacian edge detector

Sobel edge detector

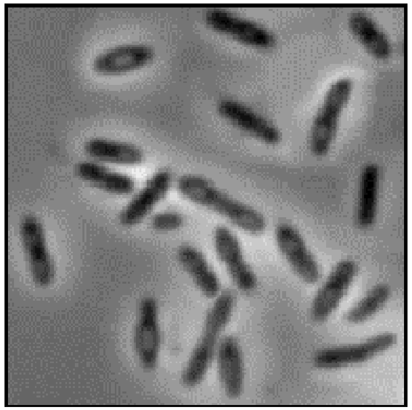


Negative mag. of Gradient
(noise-free image)



Thresholded output
(noise-free image)

Laplacian edge detector



Bacteria image

Laplacian of image

Thresholded output

Laplacian of Gaussian (LoG) edge detector

- Gaussian (G)

$$G(x, y) = e^{-\frac{x^2 + y^2}{2\sigma^2}}$$

- Laplacian of Gaussian (LoG, ∇^2G)

$$\nabla^2 G(x, y) = \frac{\partial^2 G(x, y)}{\partial x^2} + \frac{\partial^2 G(x, y)}{\partial y^2}$$

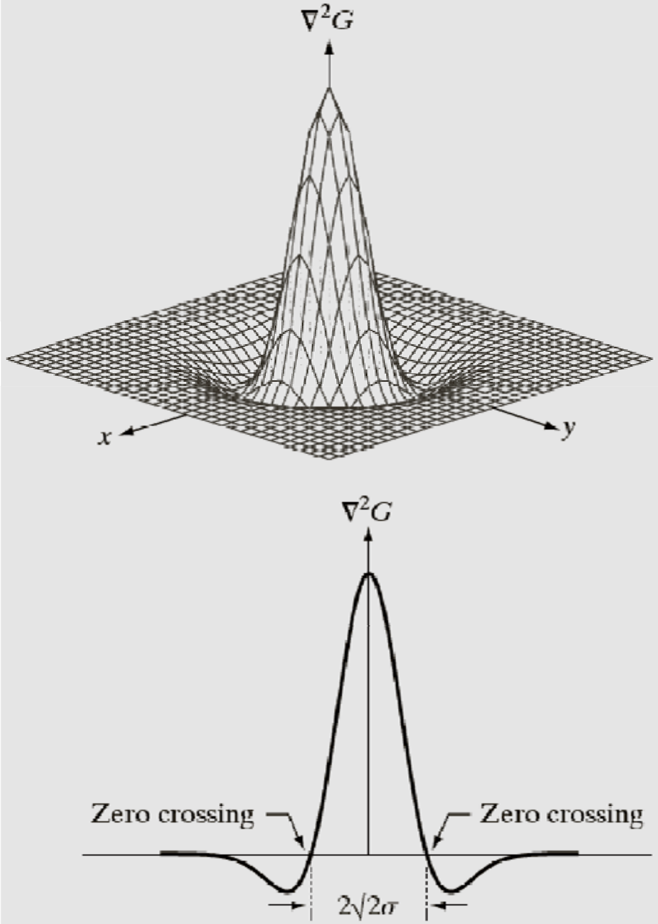
$$= \left[\frac{x^2 + y^2 - 2\sigma^2}{\sigma^4}\right] \cdot e^{-\frac{x^2 + y^2}{\sigma^2}}$$

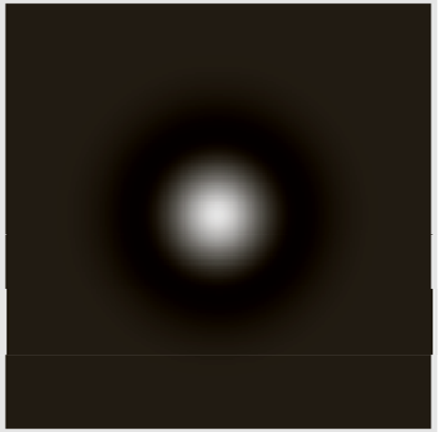
Laplacian of Gaussian (LoG)

- The mask of approximation is not unique.
  - Arbitrary size of mask
  - Tuned to act at any desired scale (large mask for blurry edges)
- How to generate a LoG mask?
  - Sampling the LoG equation directly
  - Sampling the Gaussian function and do convolution with the Laplacian mask

Laplacian of Gaussian (LoG)

- Proposed by Marr and Hildreth (1980)





0	0	−1	0	0
0	−1	−2	−1	0
−1	−2	16	−2	−1
0	−1	−2	−1	0
0	0	−1	0	0

5x5 mask approximation

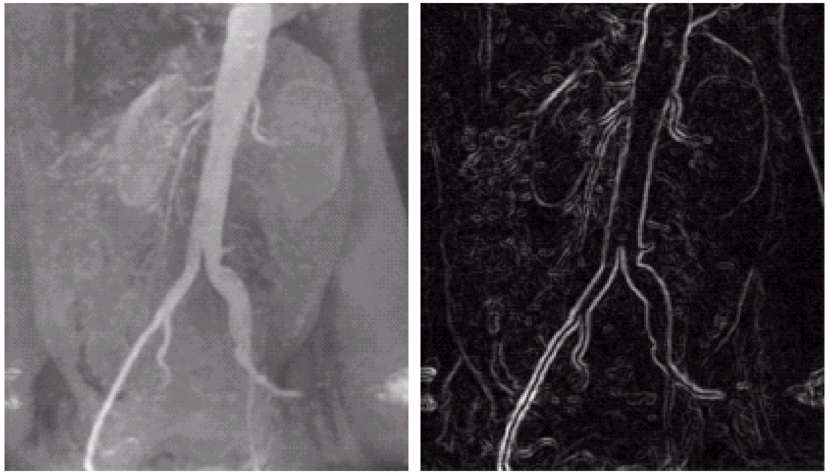
Example

a. Original image
b. Sobel Gradient

c. Spatial Gaussian smoothing function
d. Laplacian mask

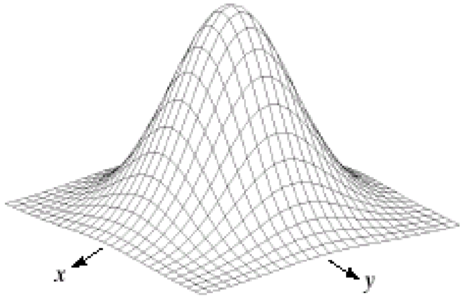
e. LoG
f. Threshold LoG
g. Zero crossing (spaghetti effect)

a



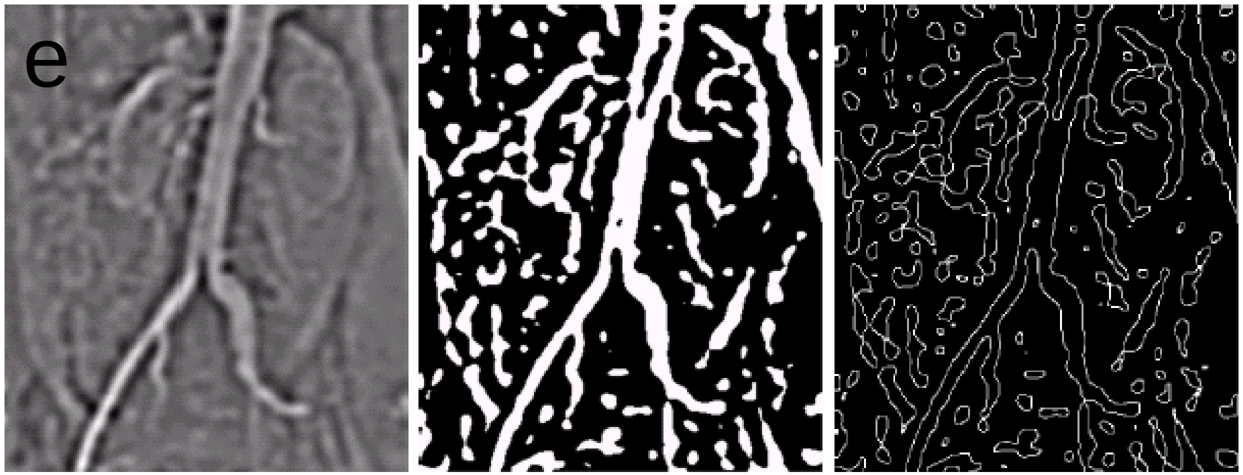
b

c



d

−1	−1	−1
−1	8	−1
−1	−1	−1



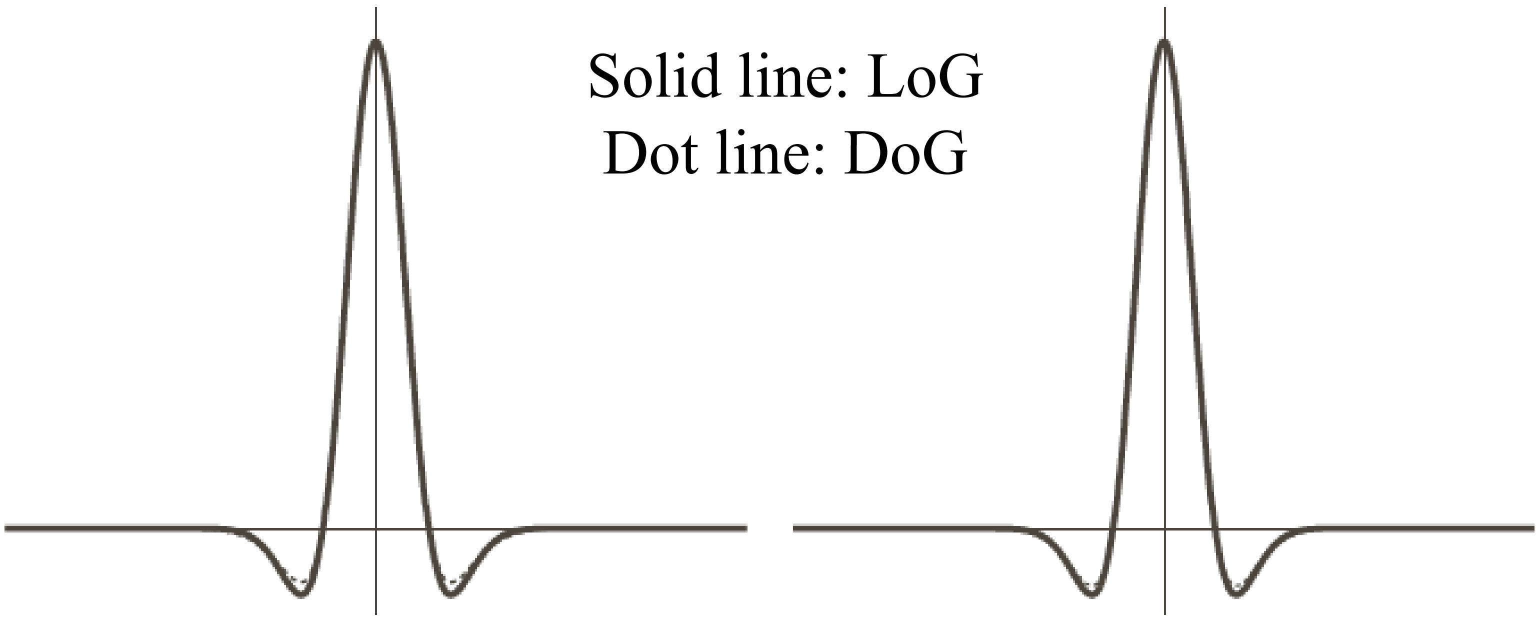
Difference of Gaussian (DoG)

- The LoG (with a standard deviation of σ) can be approximated by difference of two Gaussian distributions. ($\sigma_1 < \sigma_2$)

$$DoG(x,y) = \frac{1}{2\pi\sigma_1^2} e^{-\frac{x^2+y^2}{2\sigma_1^2}} - \frac{1}{2\pi\sigma_2^2} e^{-\frac{x^2+y^2}{2\sigma_2^2}}$$

$$\text{where} \quad \sigma^2 = \frac{\sigma_1^2\sigma_2^2}{\sigma_1^2 - \sigma_2^2} \ln\left[\frac{\sigma_1^2}{\sigma_2^2}\right]$$

Difference of Gaussian (DoG)



Standard deviation ratio = 1.75 (left) and 1.6 (right)

Properties of edge detection

- Scalar multiplication or sign of matrices plays no essential role in edge detection.
- All derivative masks here have a mean of zero.
- Sensitive to noise.
  - Morphological processing may help remove the false positives (opening) or recover the false negatives (closing).

Opening and Closing

Noisy image

After opening

After opening & closing

Still room for improvement

- Better segmentation means…
  - Low error rate
  - Edge points should be well localized
  - Single edge point response
- The Canny edge detector (1986)

The Canny edge detector

- Canny's approach
  - Smooth the input image with a Gaussian filter
  - Compute the gradient magnitude and phase images
  - Apply non-maxima suppression to the gradient magnitude image
  - Using double thresholding and connectivity analysis to detect and link edges

(See details in textbook.)

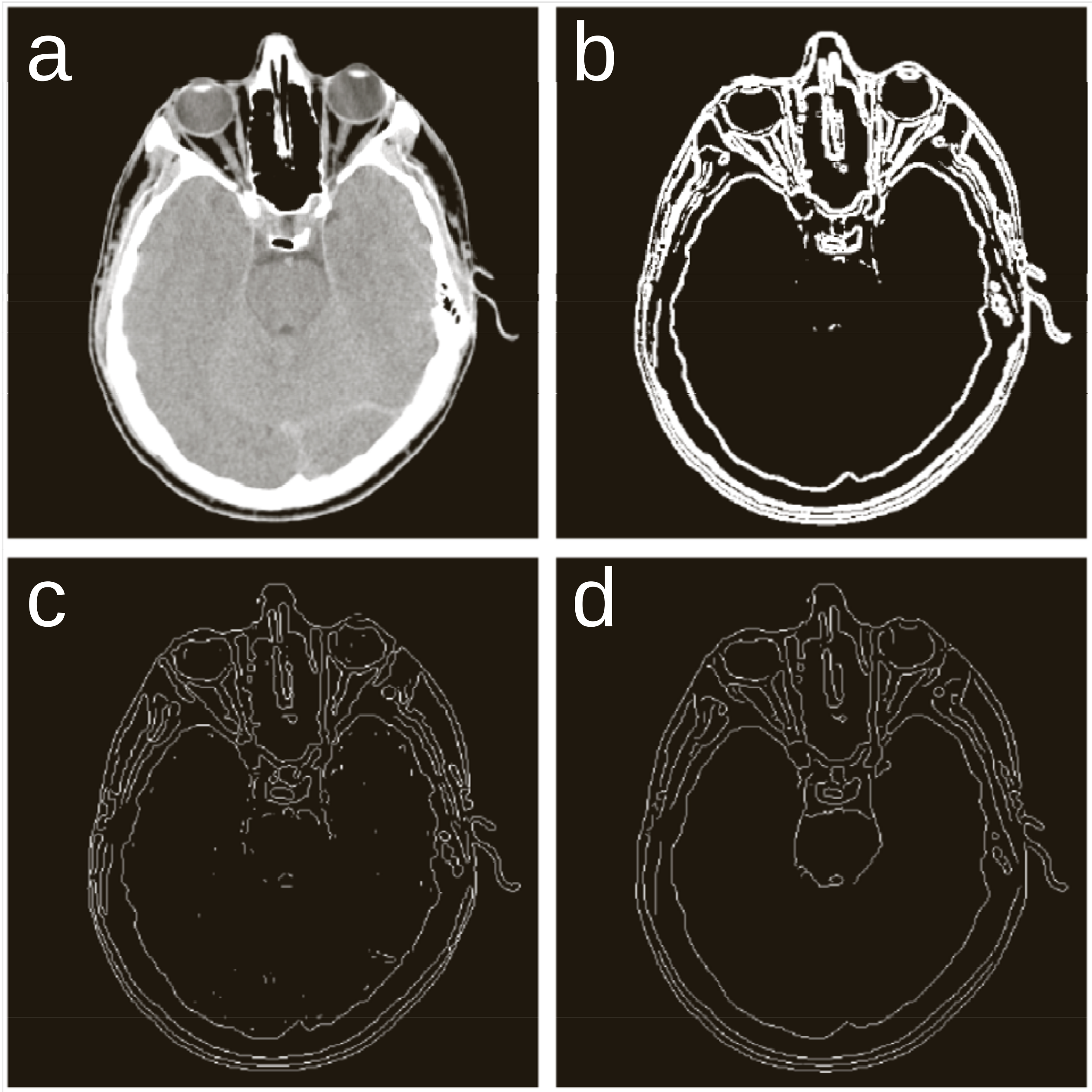
Example

a. Original image (512x512)

b. Thresholded gradient magnitude of smoothed image

c. Marr-Hildreth algorithm

d. Canny algorithm



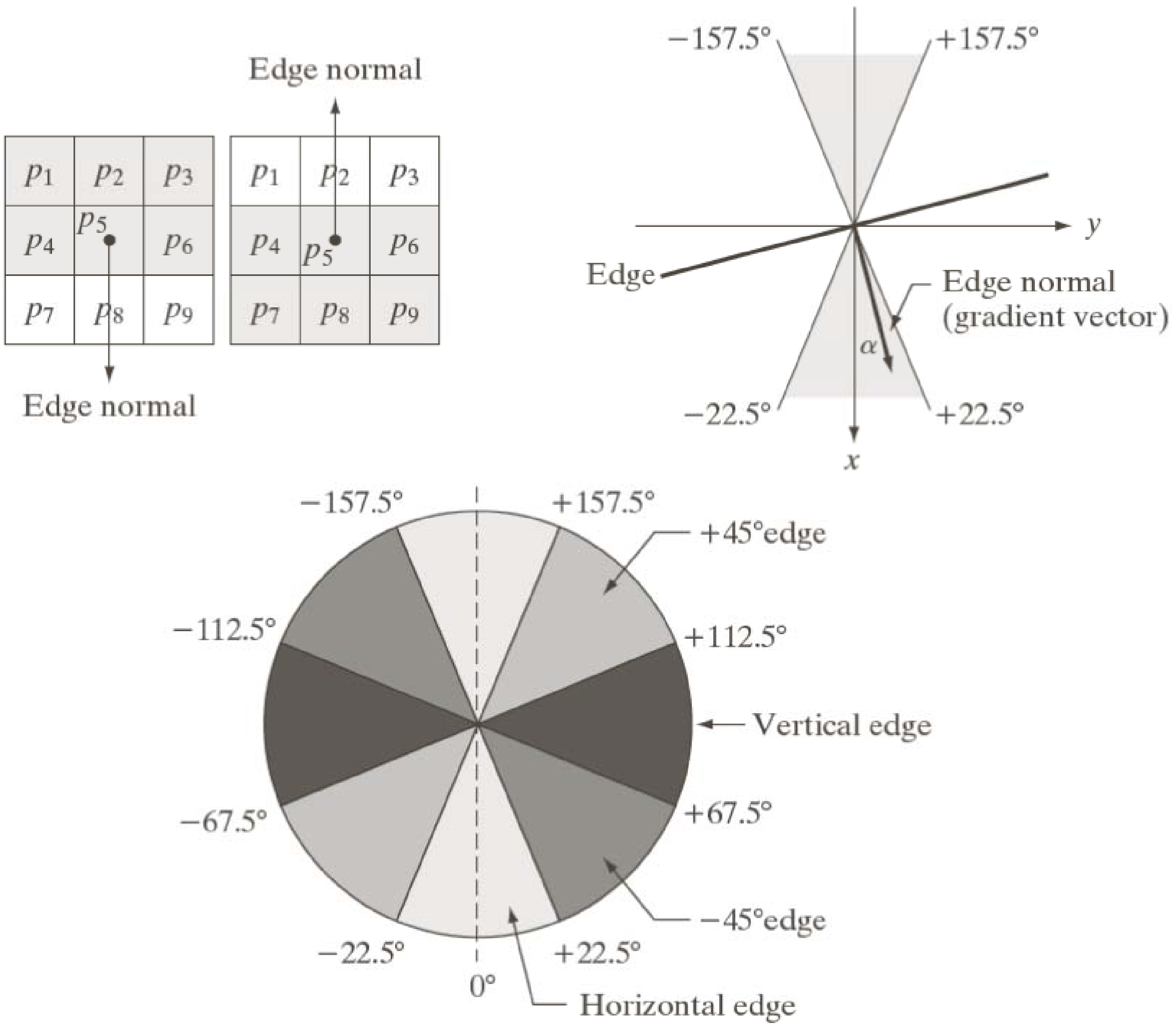
Edge Linking

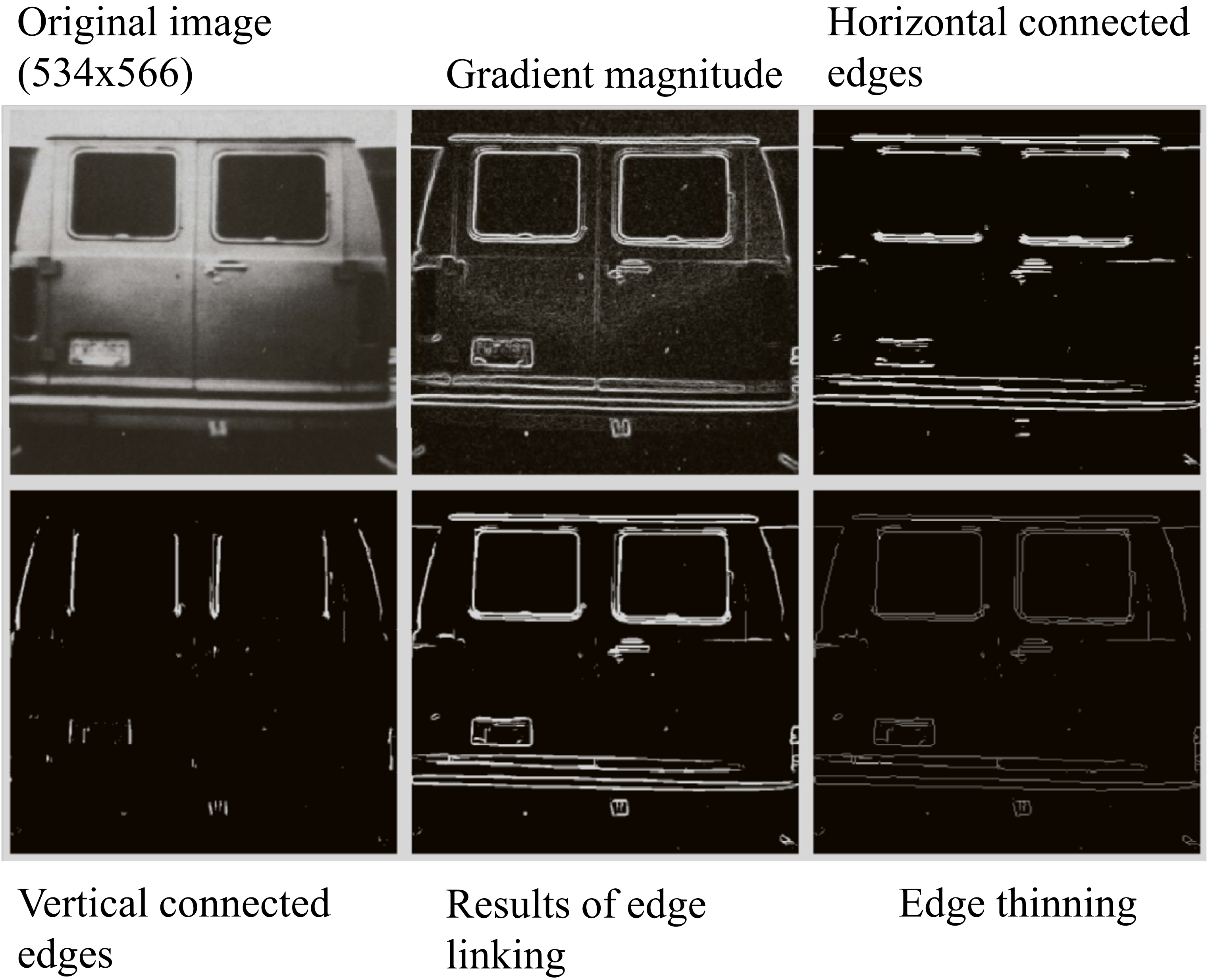
- Assemble edge pixels into meaningful edges.
- Basic approaches
  - Local Processing
  - Global Processing via the Hough Transform

Local Processing

- In a pre-defined small neighborhood (say, 3x3, 5x5), find pixel(s) similar in magnitude and angle of the gradient.
- A link is established between (x, y) and its neighbor if the previous criteria is satisfied.
- Repeat the procedure on every pixel.

Similarity in phase of gradient



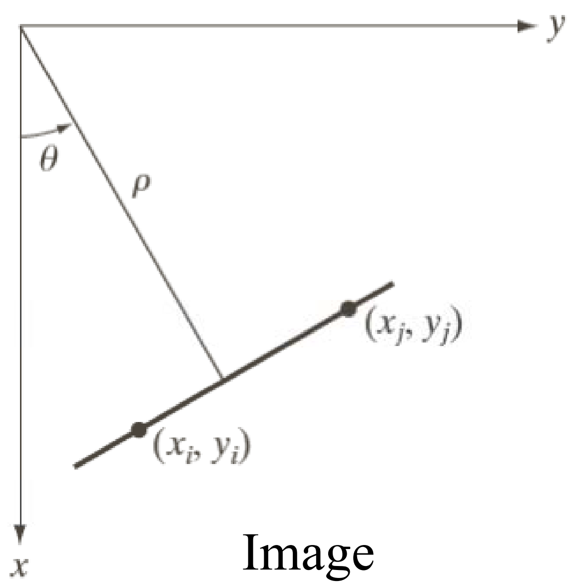

Original image
(534x566)
Gradient magnitude
Horizontal connected
edges
Vertical connected
edges
Results of edge
linking
Edge thinning

Global processing of edge linking

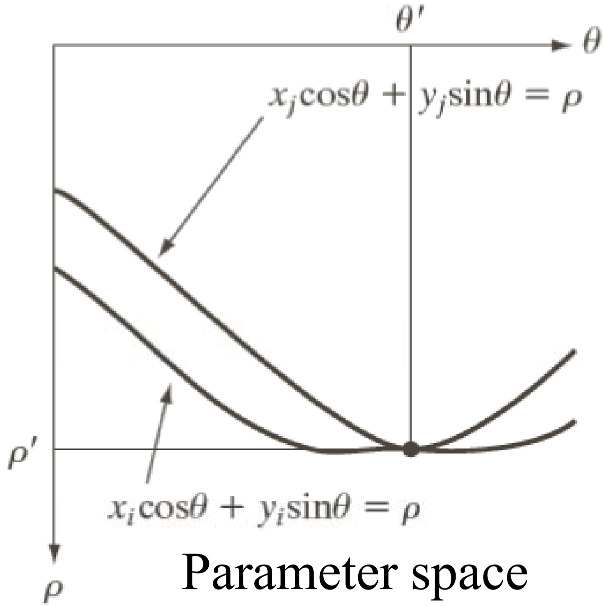
- All pixels are candidates
- Linking curves are specified based on global properties
- Hough transform
  - Transform the image into a parameter space in which the desired shape can be defined analytically.

Hough Transform: $\rho\theta$-plane

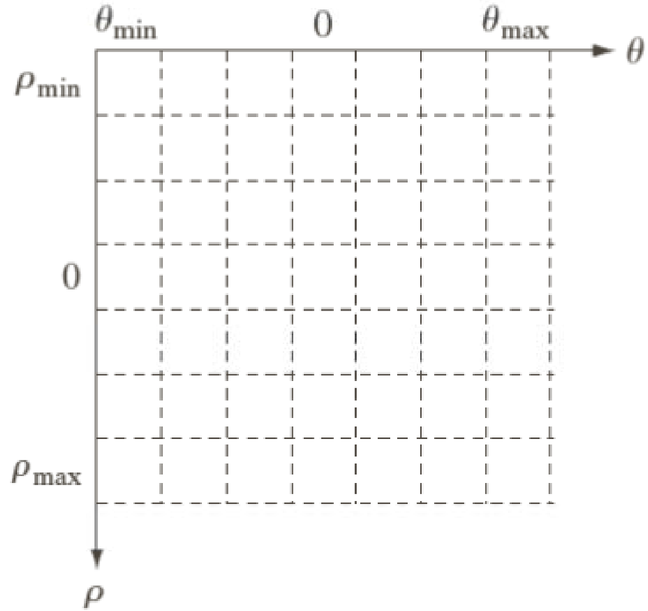
Single point in xy-plane → a sinusoidal curve in $\rho\theta$-plane
Straight line in xy-plane → single intersection point in $\rho\theta$-plane



Image



Parameter space



A line in xy-plane can be expressed as:

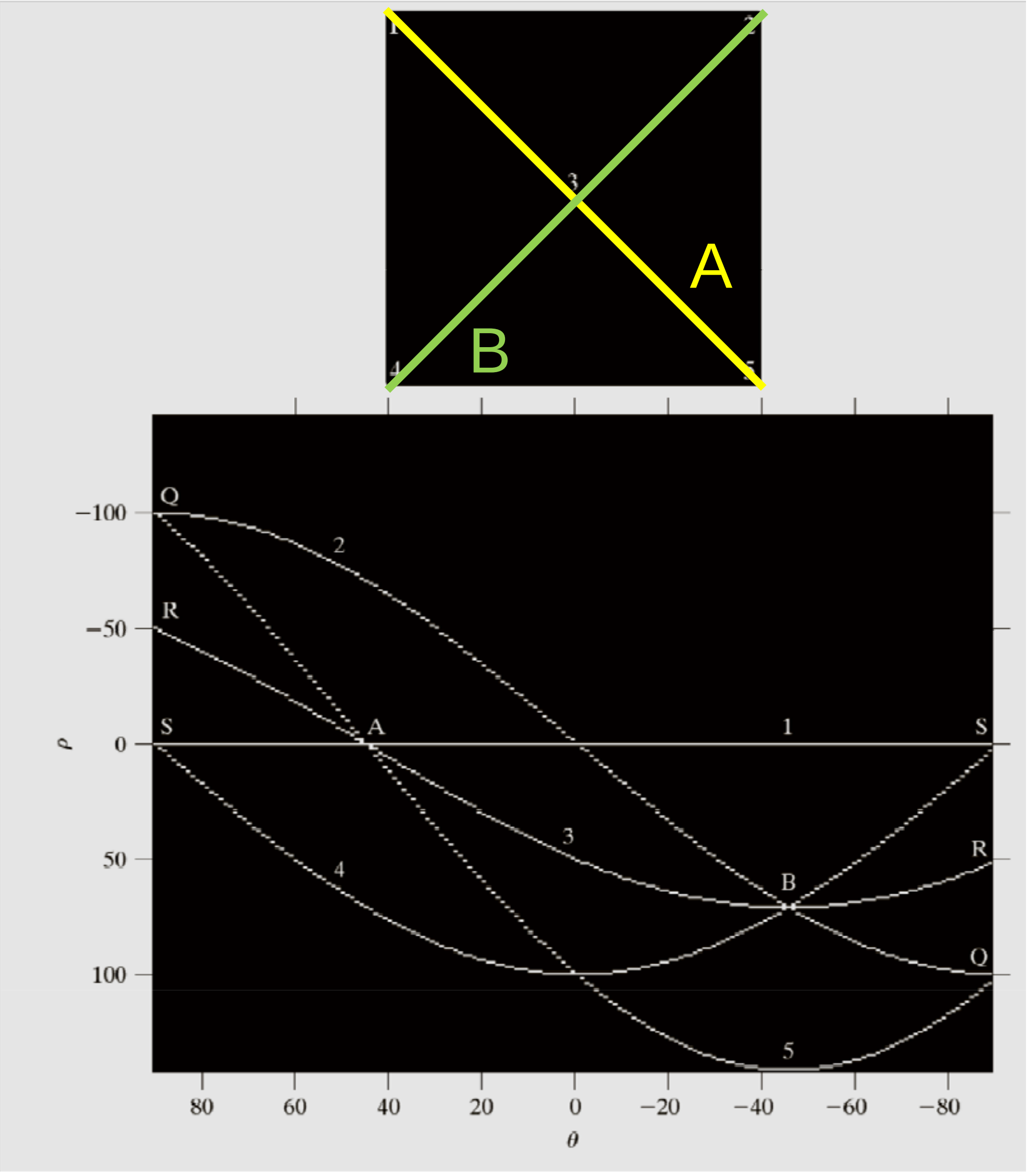
$y = ax + b$

or $x \cos\theta + y \sin\theta = \rho$

Division of the $\rho\theta$-plane into accumulator cells

Why not use a and b for classification?

Example: Hough transform



Procedure of global edge detection

1. Obtain a binary image
2. Specify sub-divisions in the $\rho\theta$-plane
3. Examine the counts of accumulator cells
   – Looking for high concentration
4. Examine the relationship (continuity) in a chosen cell and connect the gap
   – Small cell → colinearity of points

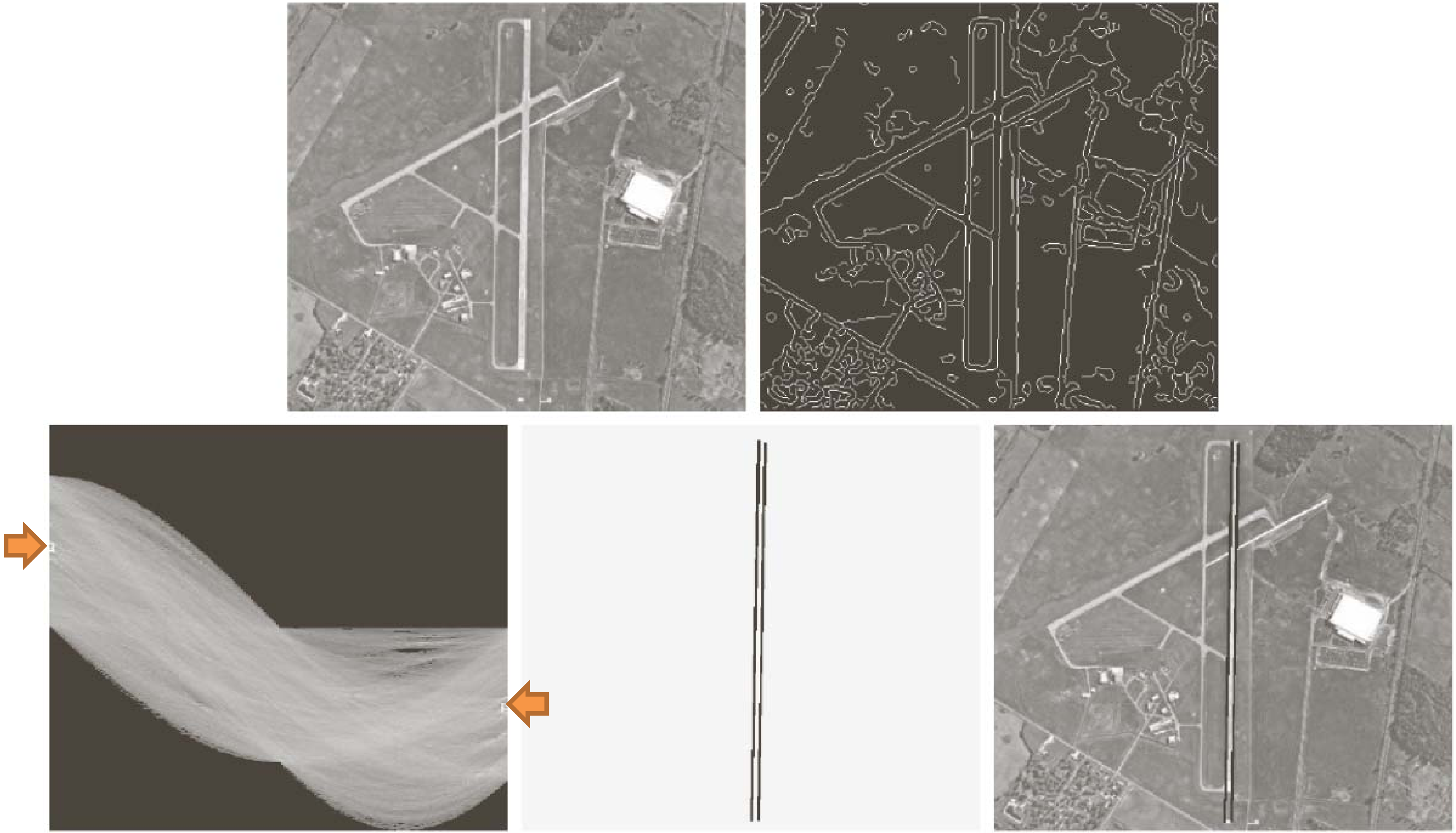
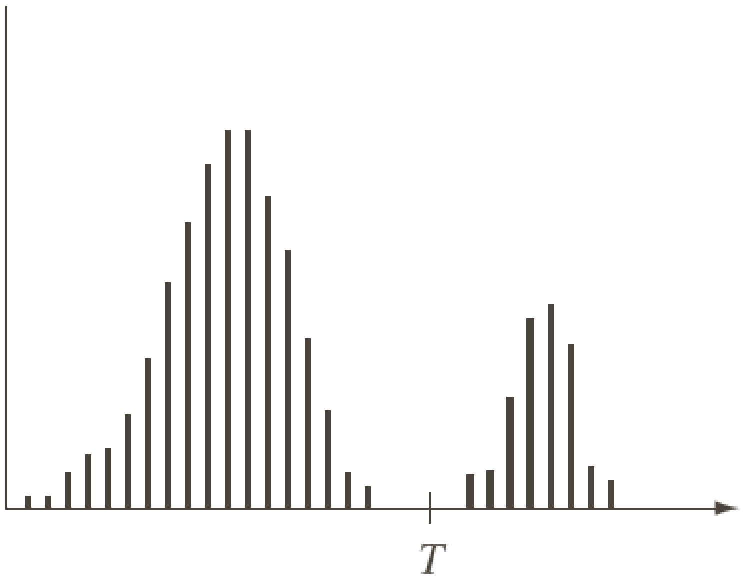


FIGURE 10.34 (a) A 502 × 564 aerial image of an airport. (b) Edge image obtained using Canny's algorithm. (c) Hough parameter space (the boxes highlight the points associated with long vertical lines). (d) Lines in the image plane corresponding to the points highlighted by the boxes). (e) Lines superimposed on the original image.

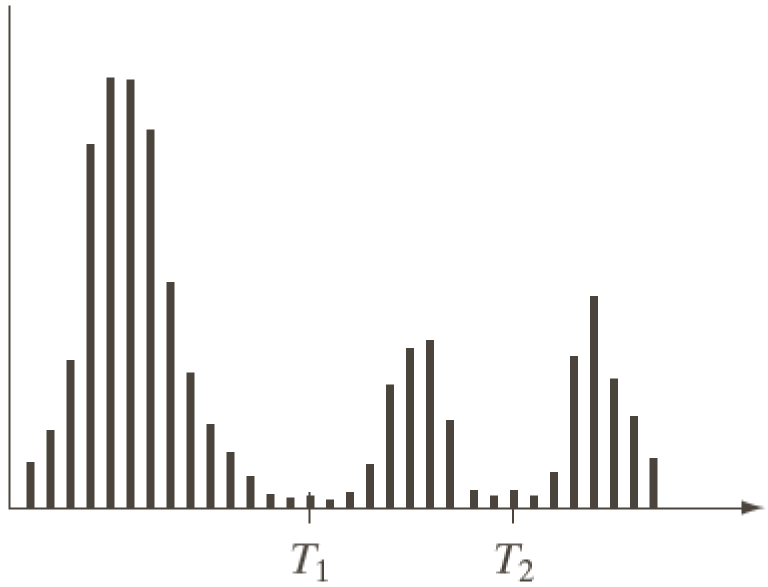
Thresholding-based method

Thresholding

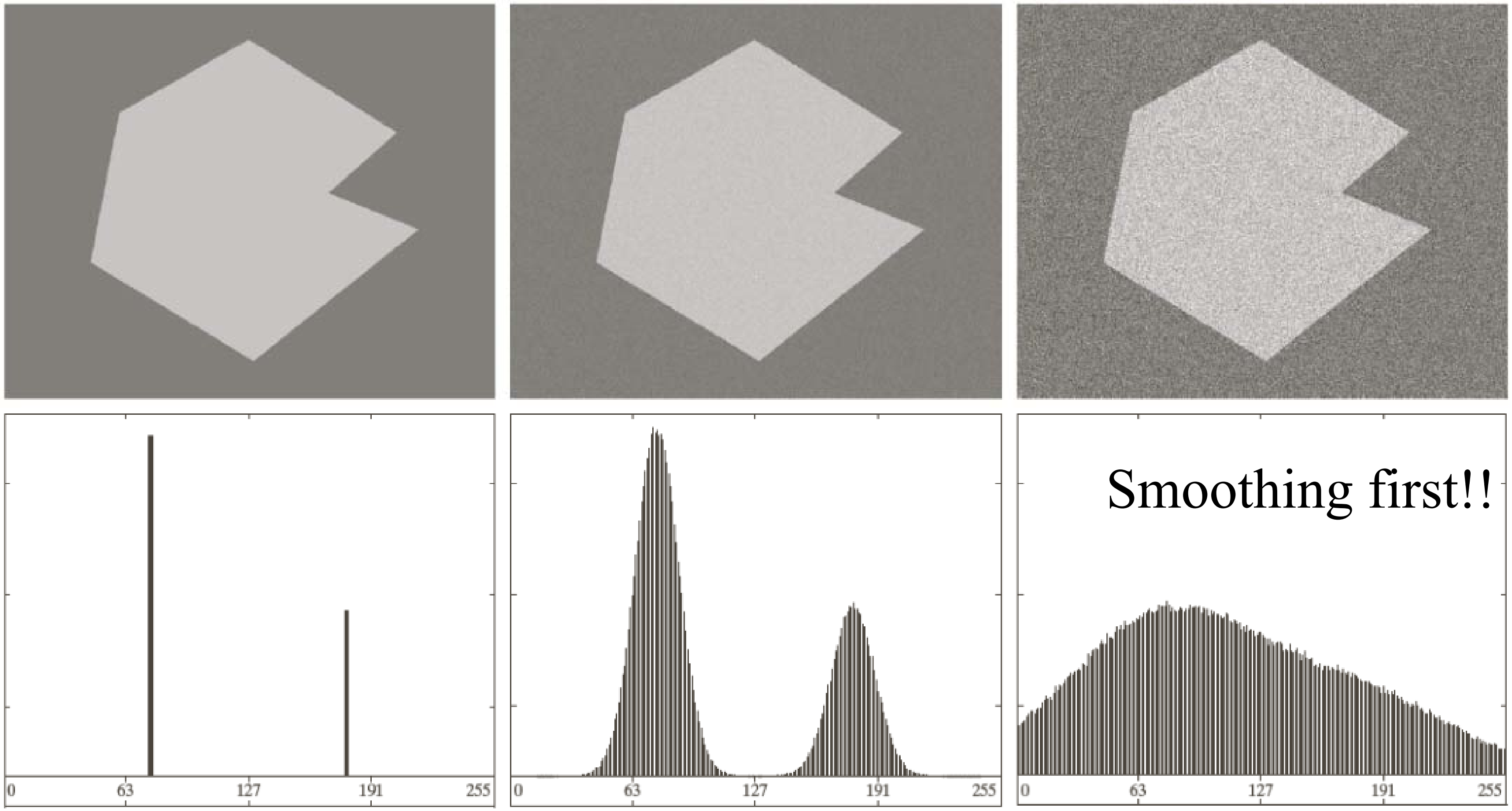
Single threshold



Dual thresholds

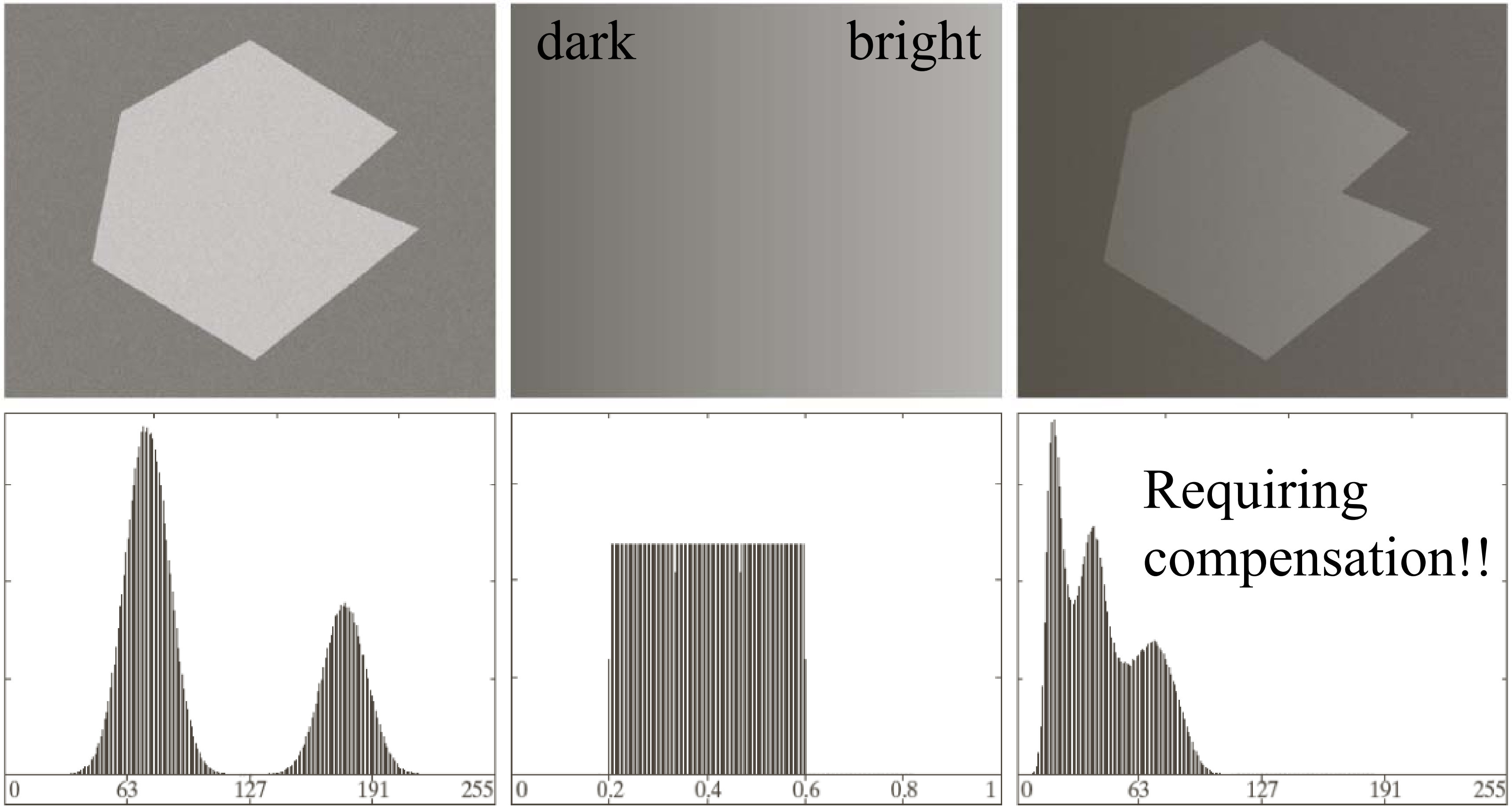


Noise in thresholding



Without noise $\mu = 0, \sigma = 10$ $\mu = 0, \sigma = 50$

Illumination in thresholding



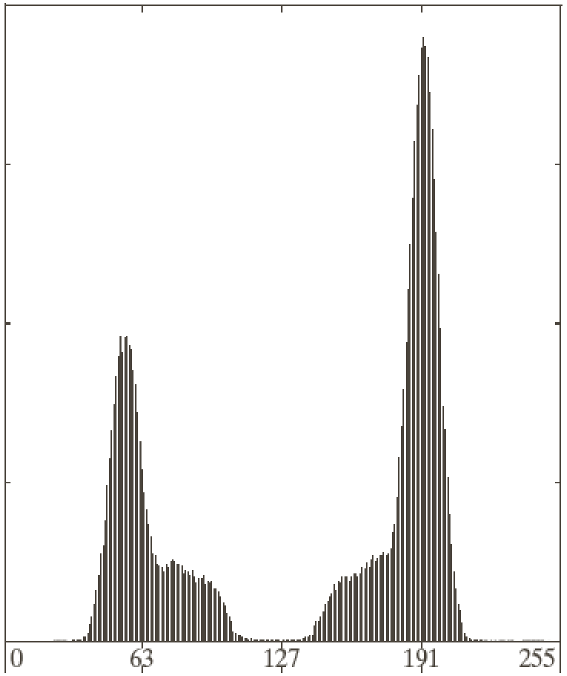
Intensity ramp

Global thresholding

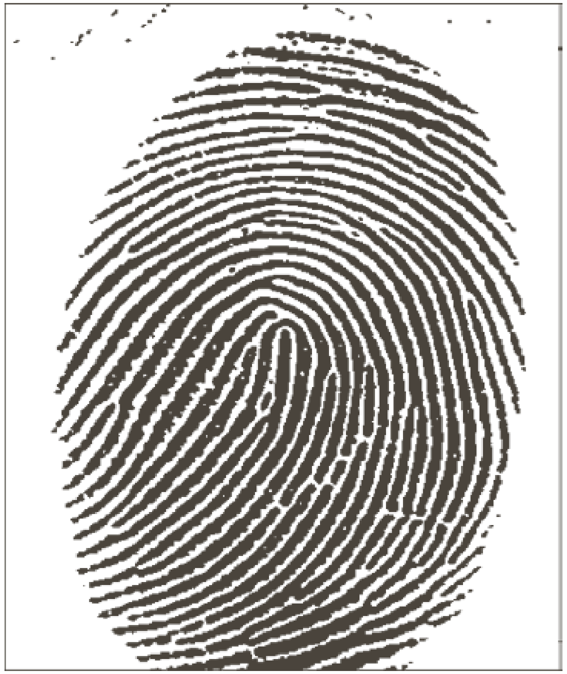
- Only gray-level values are considered.
- A simple algorithm of iteration:
  1. Select the initial value T for global thresholding.
  2. Segment the image using T.
  3. Compute the average intensity of two groups, m_1 and m_2. (intensity $< T$ and $> T$)
  4. Compute a new threshold value $T = (m_1 + m_2)/2$.
  5. Repeat steps 2-4 until ΔT is small enough.

Example: global threshold

noisy fingerprint



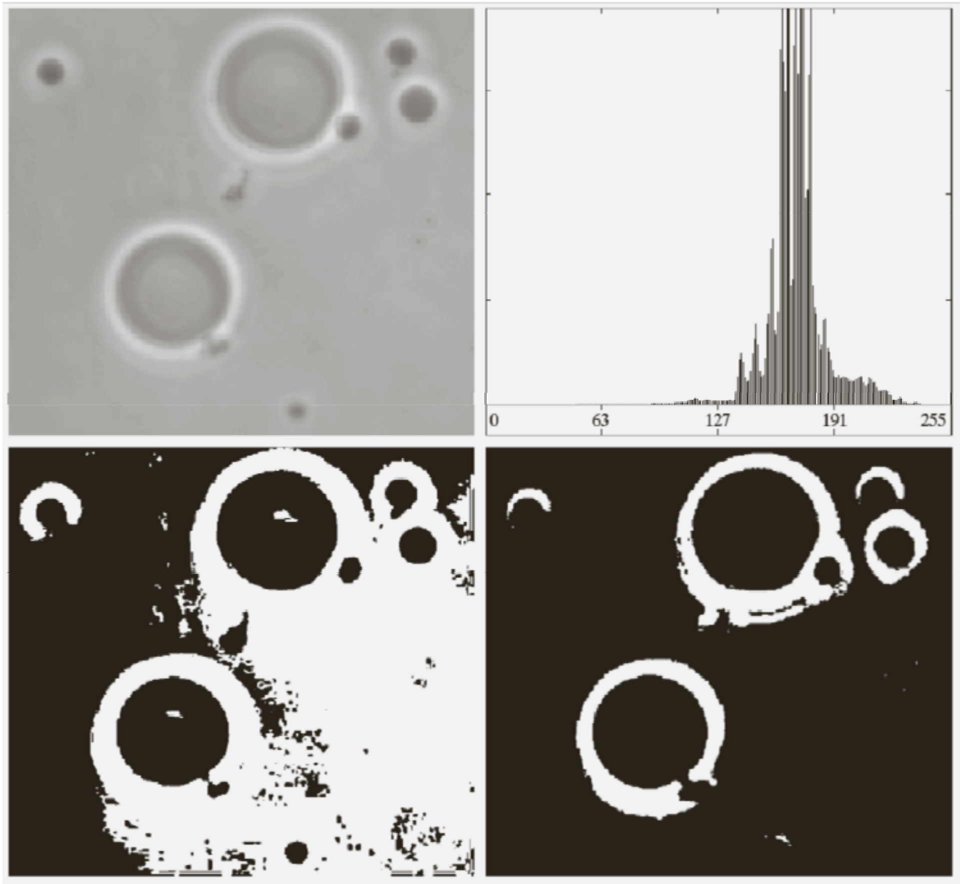
histogram



segmented results (T = 125.4 after three iterations)

Optimum global thresholding

- Basic idea: maximize the between-class variance (Ostu's method)



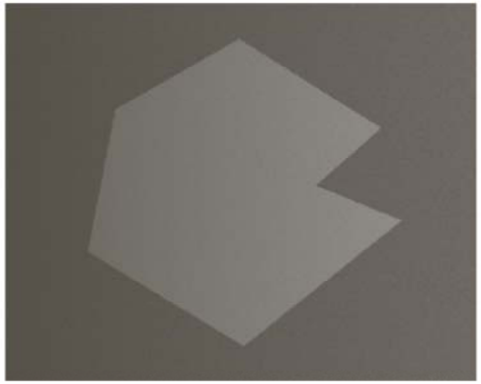
Original

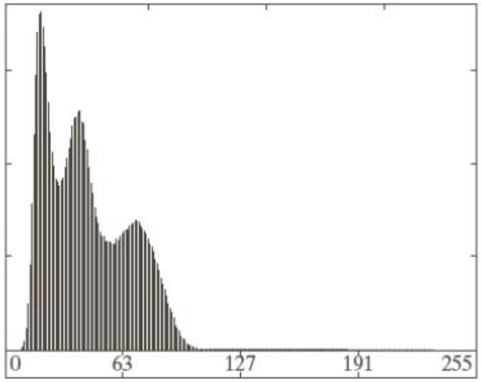
Histogram

Basic global thresholding (T = 169)

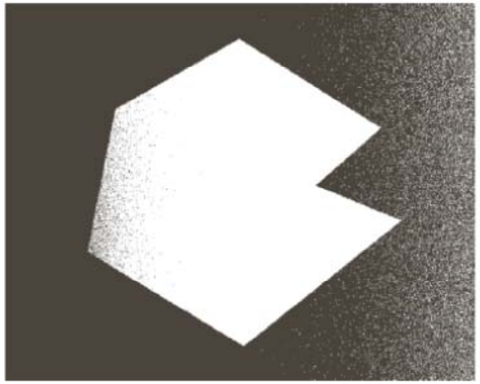
Optimum global thresholding (T = 181)

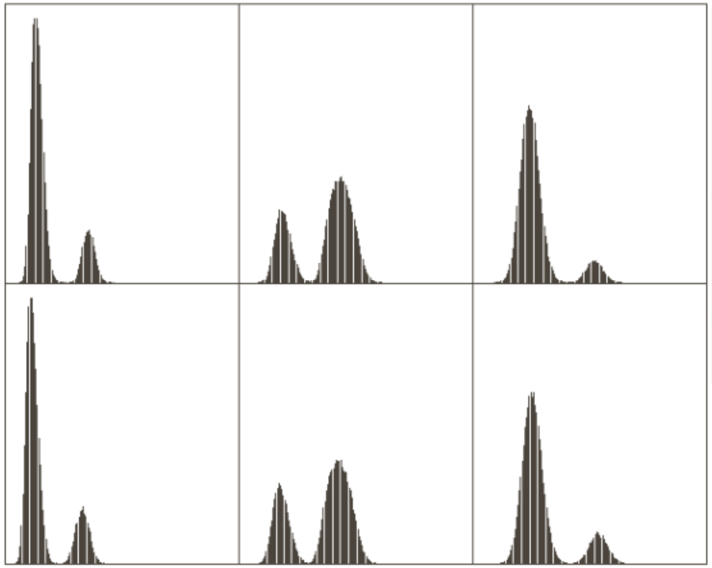
Image partitioning

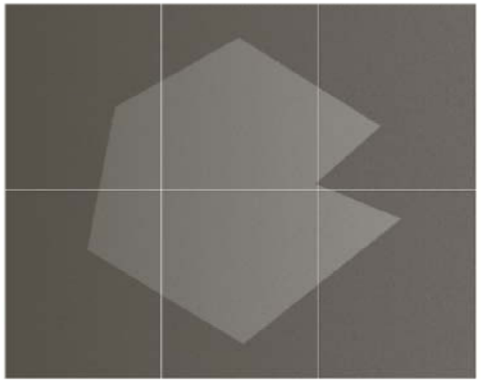




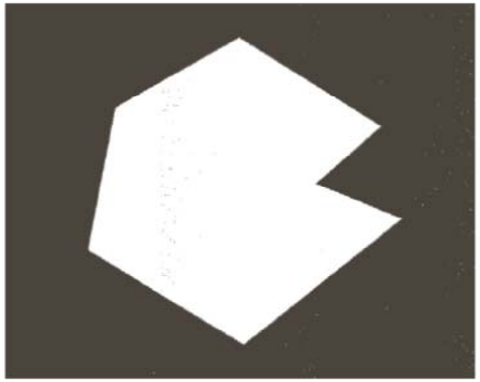
Thresholding using optimum method





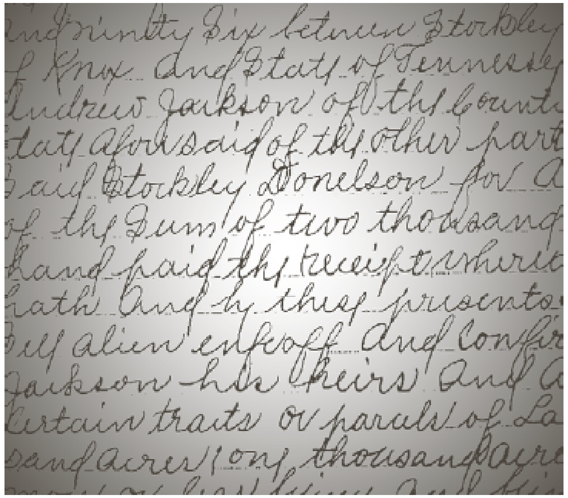


Divided into 6 sub-images and their histograms (left)

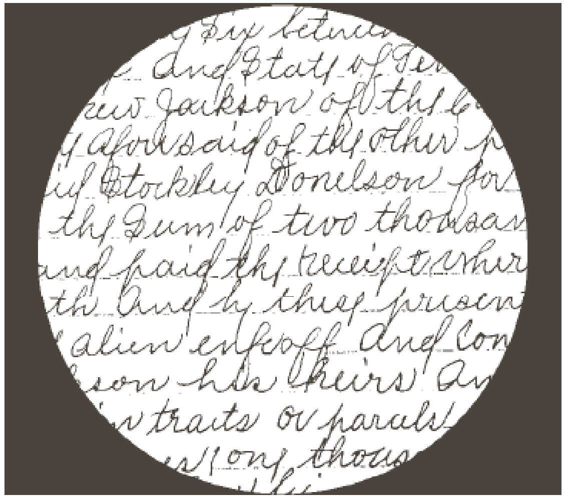


Thresholding in individual sub-image

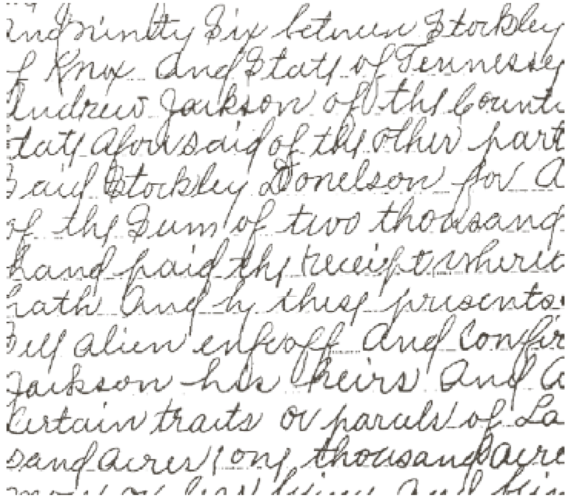
Local segmentation



Original image



Global thresholding



Local segmentation using moving window

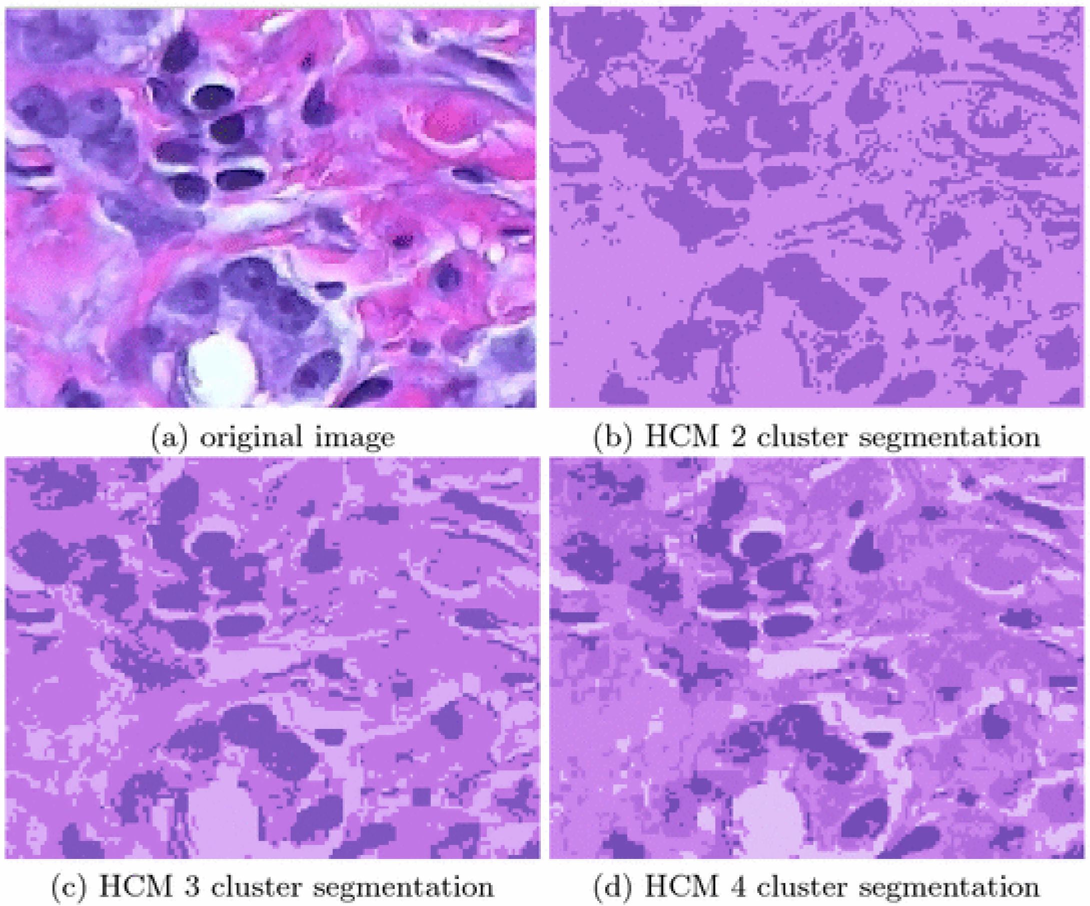
Useful in document processing!!

More extensions

- Multiple thresholds
  - k-means clustering or hard c-means clustering
  - Fuzzy c-means clustering
- Region-based growing
- Watershed segmentation

- Cortical surface rendering (GM/WM/CSF separation in brain)

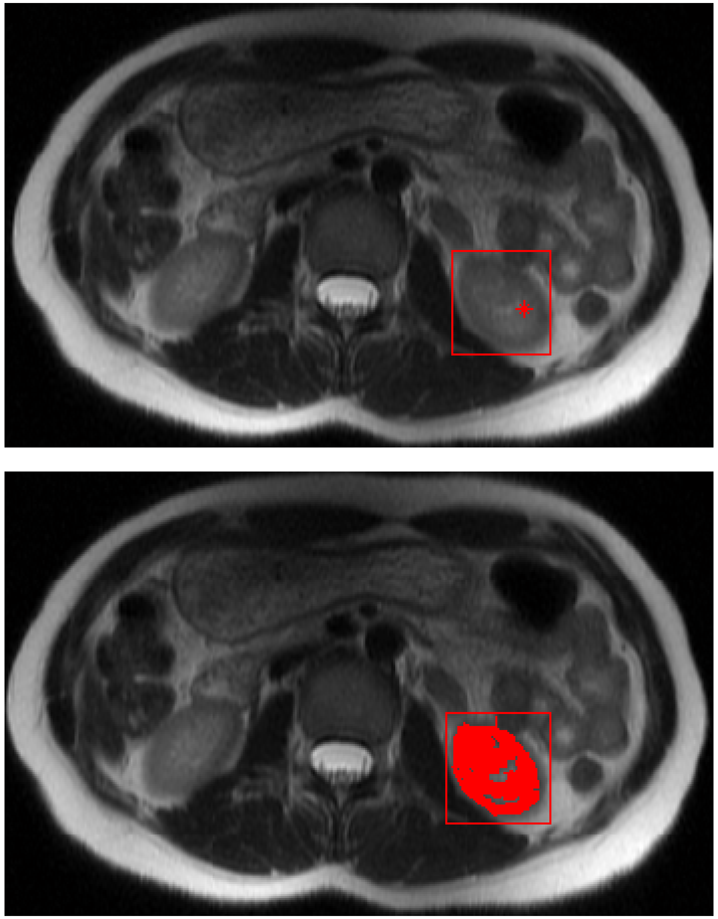
K-means clustering



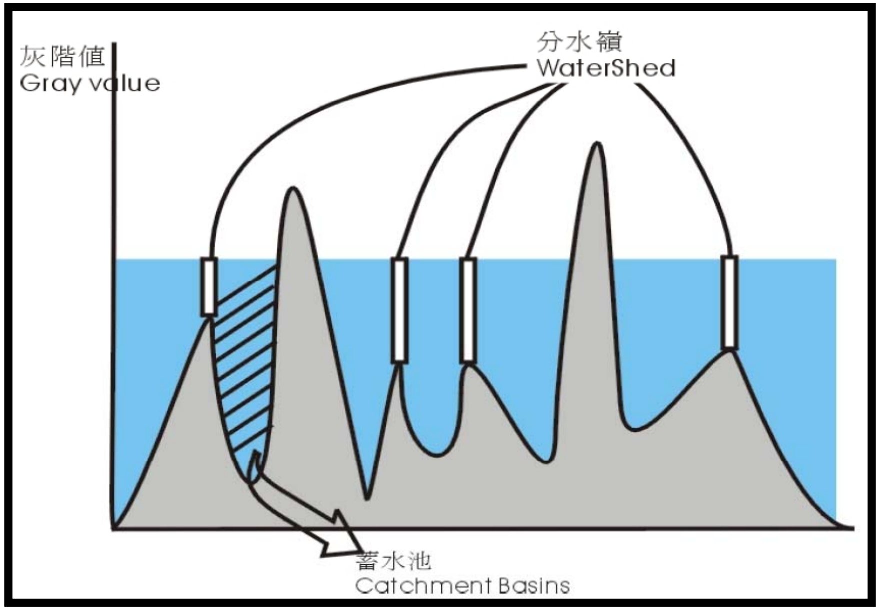
Stained tissue image (a) and segmentations based on hard c-means clustering (HCM) with 2-4 clusters

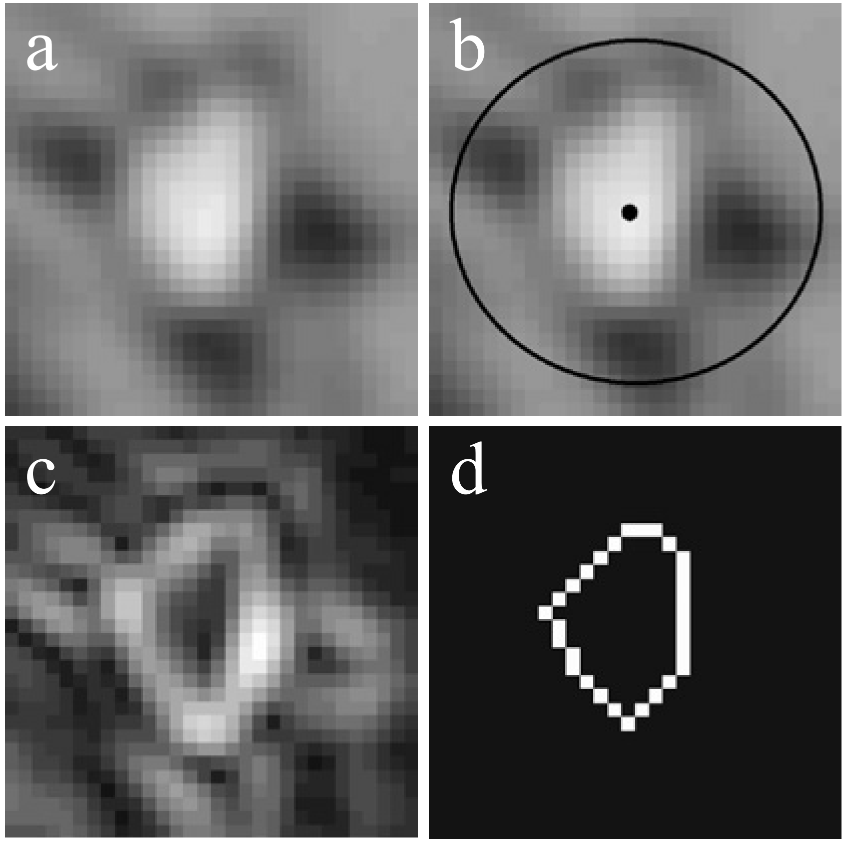
Region Growing

- The identification of region starts with a single pixel (seed).
- Intake the neighboring pixels are similar to the seed based on predefined criteria.



Watershed segmentation

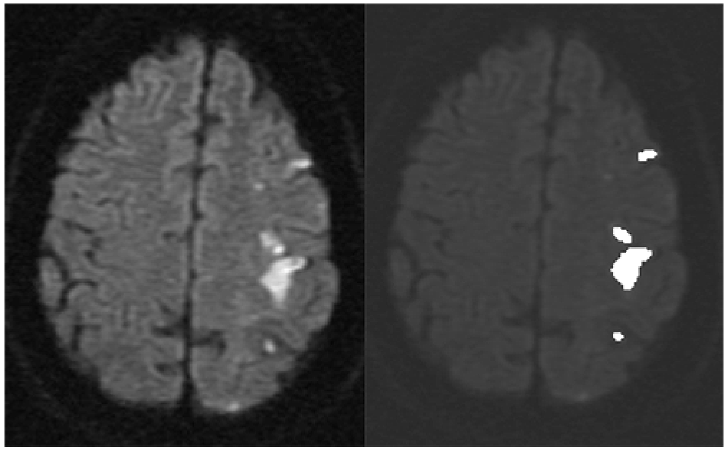




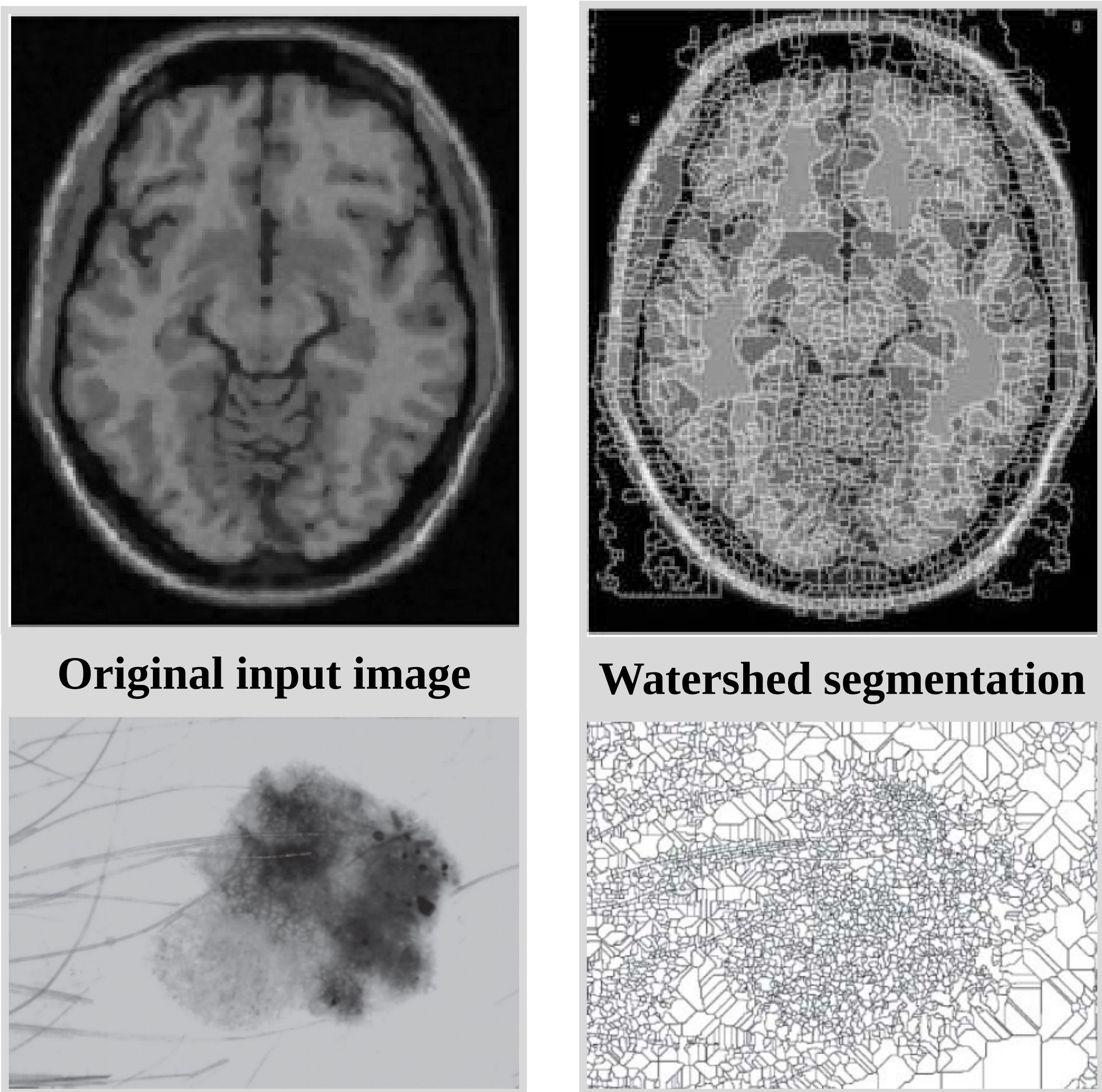
(a) Original CT image of a lymph node
(b) Prior knowledge defined by the operator
(c) Sobel edge detection of (a)
(d) Watershed lines performed on (c)

Medical image applications

- Detection of microcalcification on mammograms
- Vessel extraction on angiograms
- Classification of cells
- Lesion quantification
- Surgical planning
- More…



Challenges



Upper:
MRI brain image

Lower:
Skin lesion image

生醫影像研究方法：影像分割